

Mathematics

(Chapter – 13) (Surface Areas and Volumes)

(Class – IX)

EXERCISE 13.1

- Q.1.** A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine :
- (i) The area of the sheet required for making the box.
(ii) The cost of sheet for it, if a sheet measuring 1 m² costs Rs 20.

Sol. Here, $l = 1.5$ m, $b = 1.25$ m, $h = 65$ cm = 0.65 m.
Since the box is open at the top, it has only five faces.

(i) So, surface area of the box = $lb + 2(bh + hl)$
 $= 1.5 \times 1.25 \text{ m}^2 + 2(1.25 \times 0.65 + 0.65 \times 1.5) \text{ m}^2$
 $= 1.875 + 2(1.7875) \text{ m}^2$
 $= (1.875 + 3.575) \text{ m}^2 = 5.45 \text{ m}^2$

Hence, 5.45 m² of sheet is required **Ans.**

(ii) Cost of 1 m² of the sheet = Rs 20

\therefore cost of 5.45 m² of the sheet = Rs 20 \times 5.45 m² = **Rs 109 Ans.**

- Q.2.** The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs 7.50 per m².

Sol. Here, $l = 5$ m, $b = 4$ m, $h = 3$ m
Surface area of the walls of the room and the ceiling

$$= 2h(l + b) + lb$$
$$= [2 \times 3(5 + 4) + 5 \times 4] \text{ m}^2$$
$$= (6 \times 9 + 20) \text{ m}^2 = 74 \text{ m}^2$$

Cost of white washing = Rs 7.50 per m²

\therefore total cost of white washing the walls and the ceiling of the room
 $= \text{Rs } 74 \times 7.50 = \text{Rs } 555 \text{ Ans.}$

- Q.3.** The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs 10 per m² is Rs 15000, find the height of the hall.

Sol. Let length, breadth and height of the hall be l , b and h respectively.

Perimeter of the floor of the hall = $2(l + b) = 250$ m.

Area of the four walls of the hall = $2h(l + b)$... (i)

Also, area of the four walls of the hall = $\frac{15000}{10} \text{ m}^2$

$$= 1500 \text{ m}^2 \quad \dots \text{ (ii)}$$

From (i) and (ii), we have

$$2h(l + b) = 1500$$

$$\Rightarrow h \times 250 = 1500 \quad [\because 2(l + b) = 250]$$

$$\Rightarrow h = \frac{1500}{250} = 6$$

Hence, height of the hall is **6 m Ans.**

Q.4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2 . How many bricks of dimensions $22.5 \text{ cm} \times 10 \text{ cm} \times 7.5 \text{ cm}$ can be painted out of this container?

Sol. Here, $l = 22.5 \text{ cm}$, $b = 10 \text{ cm}$, $h = 7.5 \text{ cm}$.

$$\begin{aligned}\text{Total surface area of 1 brick} &= 2(lb + bh + hl) \\ &= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5) \text{ cm}^2 \\ &= 2(225 + 75 + 168.75) \text{ cm}^2 = 937.5 \text{ cm}^2 \\ &= \frac{937.5}{100 \times 100} \text{ m}^2 = 0.09375 \text{ m}^2.\end{aligned}$$

$$\therefore \text{required number of bricks} = \frac{9.375}{0.09375} = \mathbf{100 \text{ Ans.}}$$

Q.5. A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

- (i) Which box has the greater lateral surface area and by how much?
(ii) Which box has the smaller total surface area and by how much?

Sol. Here, $a = 10 \text{ cm}$, $l = 12.5 \text{ cm}$, $b = 10 \text{ cm}$, $h = 8 \text{ cm}$

(i) Lateral surface area of the cubical box $= 4a^2$
 $= 4 \times (10)^2 \text{ cm}^2 = 400 \text{ cm}^2$

$$\begin{aligned}\text{Lateral surface area of the cuboidal box} &= 2h(l + b) \\ &= 2 \times 8(12.5 + 10) \text{ cm}^2 \\ &= 16 \times 22.5 \text{ cm}^2 = 360 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Difference in the lateral surface areas of the two boxes} \\ &= (400 - 360) \text{ cm}^2 = 40 \text{ cm}^2.\end{aligned}$$

Hence, the cubical box has greater lateral surface area by 40 cm^2 . **Ans.**

(ii) Total surface area of the cubical box $= 6a^2$
 $= 6 \times (10)^2 \text{ cm}^2 = 600 \text{ cm}^2$

$$\begin{aligned}\text{Total surface area of the cuboidal box} &= 2(lb + bh + hl) \\ &= 2(12.5 \times 10 + 10 \times 8 + 8 \times 12.5) \text{ cm}^2 \\ &= 2(125 + 80 + 100) \text{ cm}^2 \\ &= 2 \times 305 \text{ cm}^2 = 610 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Difference in the total surface areas of the two boxes} &= (610 - 600) \text{ cm}^2 \\ &= 10 \text{ cm}^2\end{aligned}$$

Hence, the cubical box has smaller total surface area by 10 cm^2 . **Ans.**

Q.6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

- (i) What is the area of the glass?
(ii) How much of tape is needed for all the 12 edges?

Sol. Here, $l = 30 \text{ cm}$, $b = 25 \text{ cm}$, $h = 25 \text{ cm}$.

(i) Total surface area of the herbarium $= 2(lb + bh + hl)$
 $= 2(30 \times 25 + 25 \times 25 + 25 \times 30) \text{ cm}^2$
 $= 2(750 + 625 + 750) \text{ cm}^2$
 $= 2 \times 2125 \text{ cm}^2 = 4250 \text{ cm}^2$

Hence, area of the glass $= 4250 \text{ cm}^2$ **Ans.**

- (ii) A cuboid has 12 edges. These consist of 4 lengths, 4 breadths and 4 heights.

$$\begin{aligned} \therefore \text{length of the tape required} &= 4l + 4b + 4h \\ &= (4 \times 30 + 4 \times 25 + 4 \times 25) \text{ cm} \\ &= (120 + 100 + 100) \text{ cm} = \mathbf{320 \text{ cm Ans.}} \end{aligned}$$

Q.7. *Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs 4 for 1000 cm², find the cost of cardboard required for supplying 250 boxes of each kind.*

Sol. **For bigger boxes :**

$$\begin{aligned} l &= 25 \text{ cm, } b = 20 \text{ cm, } h = 5 \text{ cm} \\ \text{Total surface area of 1 bigger box} &= 2(lb + bh + hl) \\ &= 2(25 \times 20 + 20 \times 5 + 5 \times 25) \text{ cm}^2 \\ &= 2(500 + 100 + 125) \text{ cm}^2 = 1450 \text{ cm}^2 \end{aligned}$$

Area of cardboard required for overlaps

$$= 5\% \text{ of } 1450 \text{ cm}^2 = \frac{1450 \times 5}{100} \text{ cm}^2 = 72.5 \text{ cm}^2.$$

Total area of cardboard needed for 1 bigger box

$$= (1450 + 72.5) \text{ cm}^2 = 1522.5 \text{ cm}^2$$

$$\begin{aligned} \text{Total area of cardboard needed for 250 bigger boxes} &= 1522.5 \times 250 \text{ cm}^2 \\ &= 380625 \text{ cm}^2. \end{aligned}$$

For smaller boxes :

$$\begin{aligned} l &= 15 \text{ cm, } b = 12 \text{ cm, } h = 5 \text{ cm} \\ \text{Total surface area of 1 smaller box} &= 2(lb + bh + hl) \\ &= 2(15 \times 12 + 12 \times 5 + 5 \times 15) \text{ cm}^2 \\ &= 2(180 + 60 + 75) \text{ cm}^2 = 630 \text{ cm}^2 \end{aligned}$$

Area of cardboard required for overlaps

$$= 5\% \text{ of } 630 \text{ cm}^2 = \frac{630 \times 5}{100} \text{ cm}^2 = 31.5 \text{ cm}^2$$

$$\begin{aligned} \text{Total area of cardboard needed for 1 smaller box} &= (630 + 31.5) \text{ cm}^2 \\ &= 661.5 \text{ cm}^2 \end{aligned}$$

Total area of cardboard needed for 250 smaller boxes

$$= 661.5 \times 250 \text{ cm}^2 = 165375 \text{ cm}^2$$

Now, total area of cardboard needed for 500 boxes (250 bigger and 250 smaller boxes) = (380625 + 165375) cm² = 546000 cm²

Cost of 1000 cm² of cardboard = Rs 4

$$\therefore \text{Cost of } 546000 \text{ cm}^2 \text{ of cardboard} = \text{Rs } \frac{4}{1000} \times 546000 = \mathbf{\text{Rs } 2184 \text{ Ans.}}$$

Q.8. *Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?*

Sol. Here, $l = 4 \text{ m, } b = 3 \text{ m, } h = 2.5 \text{ m}$

The tarpaulin is needed to cover 5 faces only (excluding the floor)

$$\begin{aligned}\text{Surface area of the shelter} &= lb + 2(bh + hl) \\ &= 4 \times 3 \text{ m}^2 + 2(3 \times 2.5 + 2.5 \times 4) \text{ m}^2 \\ &= 12 \text{ m}^2 + 2(7.5 + 10) \text{ m}^2 \\ &= (12 + 35) \text{ m}^2 = 47 \text{ m}^2\end{aligned}$$

Hence, 47 m² of tarpaulin is required to make the shelter **Ans.**

EXERCISE 13.2

Q.1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2 . Find the diameter of the base of the cylinder.

Sol. Here, $h = 14 \text{ cm}$, curved, surface area = 88 cm^2 , $r = ?$
Curved surface area of the cylinder = $2\pi rh$

$$\begin{aligned}\Rightarrow 88 &= 2 \times \frac{22}{7} \times r \times 14 \\ \Rightarrow 88 &= 44 \times 2 \times r \\ \Rightarrow r &= \frac{88}{44 \times 2} = 1\end{aligned}$$

Hence, base diameter of the cylinder = $1 \times 2 \text{ cm} = 2 \text{ cm}$ **Ans.**

Q.2. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?.

Sol. Here, $h = 1 \text{ m}$, $r = \frac{140}{2} \text{ cm} = 70 \text{ cm} = 0.7 \text{ m}$

Total surface area of the cylinder = $2\pi r (h + r)$

$$\begin{aligned}&= 2 \times \frac{22}{7} \times 0.7 (1 + 0.7) \text{ m}^2 \\ &= 44 \times 0.1 \times 1.7 \text{ m}^2 = 7.48 \text{ m}^2\end{aligned}$$

Hence, 7.48 m^2 of sheet is required **Ans.**

Q.3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see figure). Find its.

- (i) inner curved surface area,
- (ii) outer curved surface area,
- (iii) total surface area.

Sol. Here, $h = 77 \text{ cm}$,

$$\text{Outer radius (R)} = \frac{4.4}{2} \text{ cm} = 2.2 \text{ cm},$$

$$\text{Inner radius (r)} = \frac{4}{2} \text{ cm} = 2 \text{ cm}$$

(i) Inner curved surface area of the pipe

$$\begin{aligned}&= 2\pi rh \\ &= 2 \times \frac{22}{7} \times 2 \times 77 \text{ cm}^2 \\ &= 2 \times 22 \times 22 \text{ cm}^2 = \mathbf{968 \text{ cm}^2} \text{ Ans.}\end{aligned}$$



(ii) Outer curved surface area of the pipe = $2\pi R h$
 $= 2 \times \frac{22}{7} \times 2.2 \times 77 \text{ cm}^2 = 44 \times 24.2 \text{ cm}^2$
 $= \mathbf{1064.80 \text{ cm}^2 \text{ Ans.}}$

(iii) Total surface area of the pipe = inner curved surface area + outer curved surface area + areas of the two base rings.
 $= 2\pi r h + 2\pi R h + 2\pi (R^2 - r^2)$
 $= 968 \text{ cm}^2 + 1064.80 \text{ cm}^2 + 2 \times \frac{22}{7} [(2.2)^2 - 2^2] \text{ cm}^2$
 $= 2032.80 \text{ cm}^2 + 5.28 \text{ cm}^2 = \mathbf{2038.08 \text{ cm}^2 \text{ Ans.}}$

Q.4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2 .

Sol. Radius of the roller (r) = $\frac{84}{2} \text{ cm} = 42 \text{ cm}$

Length of the roller (h) = 120 cm

Curved surface area of the roller = $2\pi r h$

$= 2 \times \frac{22}{7} \times 42 \times 120 \text{ cm}^2 = 44 \times 720 \text{ cm}^2 = 31680 \text{ cm}^2$

\therefore area covered by the roller in 1 revolution = 31680 cm^2

\therefore area covered by the roller in 500 revolutions = $31680 \times 500 \text{ cm}^2$
 $= 15840000 \text{ cm}^2$

Hence, area of the playground = $\frac{15840000}{100 \times 100} \text{ m}^2 = \mathbf{1584 \text{ m}^2 \text{ Ans.}}$

Q.5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs 12.50 per m^2 .

Sol. Here, $r = \frac{50}{2} \text{ cm} = 25 \text{ cm} = 0.25 \text{ m}$, $h = 3.5 \text{ m}$

Curved surface area of the pillar = $2\pi r h$

$= 2 \times \frac{22}{7} \times 0.25 \times 3.5 \text{ m}^2 = 5.5 \text{ m}^2$

Cost of painting $1 \text{ m}^2 = \text{Rs } 12.50$

\therefore Total cost of painting the curved surface of the pillar

$= \text{Rs } 12.50 \times 5.5 = \mathbf{\text{Rs } 68.75 \text{ Ans.}}$

Q.6. Curved surface area of a right circular cylinder is 4.4 m^2 . If the radius of the base of the cylinder is 0.7 m , find its height.

Sol. Curved surface area of the cylinder = 4.4 m^2 , $r = 0.7 \text{ m}$, $h = ?$

Curved surface area of the cylinder = $2\pi r h$.

$\Rightarrow 4.4 = 2 \times \frac{22}{7} \times 0.7 \times h$

$\Rightarrow h = \frac{4.4}{4.4} = 1$

Hence, height of the cylinder is $1 \text{ m} \text{ Ans.}$

- Q.7.** The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find
 (i) its inner curved surface area,
 (ii) the cost of plastering this curved surface at the rate of Rs 40 per m².

Sol. Here, $r = \frac{3.5 \text{ m}}{2}$, $h = 10 \text{ m}$

(i) Inner curved surface area of the well

$$= 2\pi rh = 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 10 \text{ m}^2$$

$$= 22 \times 5 \text{ m}^2 = \mathbf{110 \text{ m}^2 \text{ Ans.}}$$

(ii) Cost of plastering 1 m² = Rs 40

\therefore Cost of plastering the curved surface area of the well

$$= \text{Rs } 110 \times 40 = \mathbf{\text{Rs } 4400 \text{ Ans.}}$$

- Q.8.** In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

Sol. Here, $r = \frac{5}{2} \text{ cm} = 2.5 \text{ cm} = 0.025 \text{ m}$, $h = 28 \text{ m}$.

Total radiating surface in the system = total surface area of the cylinder

$$= 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 0.025 (28 + 0.025) \text{ m}^2$$

$$= \frac{44 \times 0.025 \times 28.025}{7} \text{ m}^2 = \mathbf{4.4 \text{ m}^2 \text{ (approx) Ans.}}$$

- Q.9.** Find

(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) how much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank.

Sol. Here, $r = \frac{4.2}{2} \text{ m} = 2.1 \text{ m}$, $h = 4.5 \text{ m}$

(i) Curved surface area of the storage tank = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.1 \times 4.5 \text{ m}^2 = \mathbf{59.4 \text{ m}^2 \text{ Ans.}}$$

(ii) Total surface area of the tank = $2\pi r (h + r)$

$$= 2 \times \frac{22}{7} \times 2.1 (4.5 + 2.1) \text{ m}^2$$

$$= 44 \times 0.3 \times 6.6 \text{ m}^2 = 87.12 \text{ m}^2$$

Let the actual area of steel used be $x \text{ m}^2$.

$$\text{Area of steel wasted} = \frac{1}{12} \text{ of } x \text{ m}^2 = \frac{x}{12} \text{ m}^2. \quad \dots \text{ (i)}$$

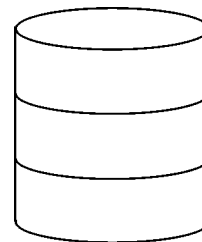
$$\therefore \text{ area of the steel used in the tank} = \left(x - \frac{x}{12} \right) \text{ m}^2 = \frac{11}{12} x \text{ m}^2$$

$$\Rightarrow 87.12 = \frac{11}{12} x$$

$$\Rightarrow x = \frac{87.12 \times 12}{11} = 95.04 \text{ m}^2$$

Hence, 95.04 m² of steel was actually used **Ans.**

Q.10. In the figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.



Sol. Here, $r = \frac{20}{2}$ cm = 10 cm

Height = 30 cm

Circumference of the base of the frame = $2\pi r$

$$= 2\pi \times 10 \text{ cm} = 20\pi \text{ cm}$$

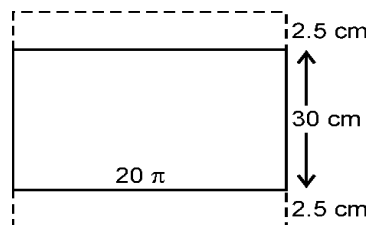
Height of the frame = 30 cm

Height of the cloth needed for covering the frame (including the margin) = $(30 + 2.5 + 2.5)$ cm = 35 cm

Also, breadth of the cloth = circumference of the base of the frame.

\therefore Area of the cloth required for covering the lampshade = length \times breadth

$$= 35 \times 20\pi \text{ cm}^2 = 35 \times 20 \times \frac{22}{7} \text{ cm}^2 = \mathbf{2200 \text{ cm}^2 \text{ Ans.}}$$



Q.11. The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

Sol. Here, $r = 3$ cm, $h = 10.5$ cm

The penholders have only one base i.e., these are open at one end.

Total surface area of 1 penholder

$$= 2\pi rh + \pi r^2 = \pi r (2h + r)$$

$$= \frac{22}{7} \times 3 (2 \times 10.5 + 3) \text{ cm}^2$$

$$= \frac{22}{7} \times 3 \times 24 \text{ cm}^2$$

Total surface area of 35 penholders = $\frac{22}{7} \times 3 \times 24 \times 35 \text{ cm}^2 = 7920 \text{ cm}^2$

Hence, 7920 cm^2 of cardboard is needed **Ans.**

EXERCISE 13.3

Q.1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. find its curved surface area.

Sol. Here, $r = \frac{10.5}{2}$ cm = 5.25 cm, $l = 10$ cm.

Curved surface area of the cone = $\pi r l$

$$= \frac{22}{7} \times 5.25 \times 10 \text{ cm}^2 = \mathbf{165 \text{ cm}^2 \text{ Ans.}}$$

Q.2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Sol. Here, $l = 21$ m, $r = \frac{24}{2}$ m = 12 m

Total surface area of the cone = $\pi r(l + r)$

$$= \frac{22}{7} \times 12 (21 + 12) \text{ m}^2$$

$$= \frac{22}{7} \times 12 \times 33 \text{ m}^2 = \mathbf{1244.57 \text{ m}^2 \text{ Ans.}}$$

Q.3. Curved surface area of a cone is 308 cm² and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.

Sol. Here, $l = 14$ cm, curved surface area = 308 cm², $r = ?$

(i) Curved surface area of the cone = $\pi r l$

$$\Rightarrow 308 = \frac{22}{7} \times r \times 14$$

$$\Rightarrow r = \frac{308}{22 \times 2} = 7$$

Hence, base radius of the cone = **7 cm.**

(ii) Total surface area of the cone = $\pi r (l + r)$

$$= \frac{22}{7} \times 7 (14 + 7) \text{ cm}^2 = 22 \times 21 \text{ cm}^2 = \mathbf{462 \text{ cm}^2 \text{ Ans.}}$$

Q.4. A conical tent is 10 m high and the radius of its base is 24 m. Find (i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m² canvas is Rs 70.

Sol. Here, $h = 10$ m, $r = 24$ m

$$\begin{aligned} \text{(i) We have, } l^2 &= h^2 + r^2 \\ &= (10)^2 + (24)^2 \\ &= 100 + 576 = 676 \end{aligned}$$

$$\Rightarrow l = \sqrt{676} = \mathbf{26 \text{ m Ans.}}$$

(ii) Curved surface area of the tent = $\pi r l$

$$= \frac{22}{7} \times 24 \times 26 \text{ m}^2$$

Cost of 1 m² canvas = Rs 70

$$\begin{aligned} \therefore \text{Cost of } \frac{22}{7} \times 24 \times 26 \text{ m}^2 \text{ of canvas} &= \text{Rs } 70 \times \frac{22}{7} \times 24 \times 26 \\ &= \mathbf{\text{Rs } 137280 \text{ Ans.}} \end{aligned}$$

Q.5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 29 cm (use $\pi = 3.14$)

Sol. Here $h = 8$ m, $r = 6$ m

$$\begin{aligned} \text{We have, } l^2 &= \sqrt{r^2 + h^2} \\ &= \sqrt{36 + 64} = \sqrt{100} = 10 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Curved surface area of the tent} &= \pi r l \\ &= 3.14 \times 6 \times 10 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{required length of tarpaulin} &= \frac{3.14 \times 6 \times 10}{3} \text{ m} + 20 \text{ cm} \\ &= 62.8 \text{ m} + 0.2 \text{ m} = \mathbf{63 \text{ m Ans.}} \end{aligned}$$

Q.6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white washing its curved surface at the rate of Rs 210 per 100 m².

Sol. Here, $l = 25$ m, $r = \frac{14}{2}$ m = 7 m

$$\begin{aligned} \text{Curved surface area of the tomb} &= \pi r l \\ &= \frac{22}{7} \times 7 \times 25 \text{ m}^2 = 550 \text{ m}^2 \end{aligned}$$

$$\text{Cost of white washing } 100 \text{ m}^2 = \text{Rs } 210$$

$$\therefore \text{Cost of white washing } 550 \text{ m}^2 = \text{Rs } \frac{210}{100} \times 550 = \mathbf{\text{Rs } 1155 \text{ Ans.}}$$

Q.7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Sol. Here, $r = 7$ cm, $h = 24$ cm

$$\begin{aligned} \text{We have, } l &= \sqrt{h^2 + r^2} = \sqrt{(24)^2 + 7^2} \\ &= \sqrt{576 + 49} = \sqrt{625} = 25 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Total curved surface area of 1 cap} &= \pi r l \\ &= \frac{22}{7} \times 7 \times 25 \text{ cm}^2 = 550 \text{ cm}^2 \end{aligned}$$

$$\text{Area of sheet required to make 10 such caps} = 10 \times 550 \text{ cm}^2 = \mathbf{5500 \text{ cm}^2 \text{ Ans.}}$$

Q.8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs 12 per m², what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

Sol. Here, $r = \frac{40}{2}$ cm = 20 cm = 0.20 m, $h = 1$ m

$$l = \sqrt{h^2 + r^2} = \sqrt{1^2 + (0.2)^2} = \sqrt{1.04} = 1.02 \text{ m}$$

Curved surface area of 1 cone = πrl

Curved surface area of 50 cones = $50 \times 3.14 \times 0.2 \times 1.02 \text{ m}^2$
= 32.028 m^2

Cost of painting an area of $1 \text{ m}^2 = \text{Rs } 12$

\therefore Cost of painting an area of $32.028 \text{ m}^2 = \text{Rs } 12 \times 32.028$
= **Rs 384.34 (approx) Ans.**

EXERCISE 13.4

Q.1. Find the surface area of a sphere of radius :

(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm

Sol. (i) $r = 10.5$ cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10.5)^2 \text{ cm}^2$$

$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5 \text{ cm}^2 = \mathbf{1386 \text{ cm}^2 \text{ Ans.}}$$

(ii) $r = 5.6$ cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (5.6)^2 \text{ cm}^2$$

$$= 4 \times \frac{22}{7} \times 5.6 \times 5.6 \text{ cm}^2 = \mathbf{394.24 \text{ cm}^2 \text{ Ans.}}$$

(iii) $r = 14$ cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2$$

$$= 88 \times 28 \text{ cm}^2 = \mathbf{2464 \text{ cm}^2 \text{ Ans.}}$$

Q.2. Find the surface area of sphere of a diameter :

(i) 14 cm (ii) 21 cm (iii) 3.5 m

Sol. (i) $r = \frac{14}{2}$ cm = 7 cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 7^2 \text{ cm}^2$$

$$= 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$88 \times 7 \text{ cm}^2 = \mathbf{616 \text{ cm}^2 \text{ Ans.}}$$

(ii) $r = \frac{21}{2}$ cm = 10.5 cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10.5)^2 \text{ cm}^2$$

$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5 \text{ cm}^2 = \mathbf{1386 \text{ cm}^2 \text{ Ans.}}$$

(iii) $r = \frac{3.5}{2}$ m = 1.75 m

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (1.75)^2 \text{ m}^2$$

$$= 4 \times \frac{22}{7} \times 1.75 \times 1.75 \text{ m}^2 = \mathbf{38.5 \text{ m}^2 \text{ Ans.}}$$

Q.3. Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$)

Sol. $r = 10$ cm

$$\begin{aligned}\text{Total surface area of the hemisphere} &= 3\pi r^2 \\ &= 3 \times 3.14 \times (10)^2 \text{ cm}^2 \\ &= 3 \times 3.14 \times 100 \text{ cm}^2 = \mathbf{942 \text{ cm}^2} \text{ Ans.}\end{aligned}$$

Q.4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Sol. When $r = 7$ cm

$$\begin{aligned}\text{Surface area of the balloon} &= 4\pi r^2 \\ &= 4 \times \pi \times 7 \times 7 \text{ cm}^2\end{aligned}$$

When $R = 14$ cm :

$$\begin{aligned}\text{Surface area of the balloon} &= 4\pi R^2 \\ &= 4 \times \pi \times 14 \times 14 \text{ cm}^2\end{aligned}$$

Required ratio of the surface areas of the balloon

$$= \frac{4 \times \pi \times 7 \times 7}{4 \times \pi \times 14 \times 14} = \frac{1}{4} = \mathbf{1 : 4} \text{ Ans.}$$

Q.5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2 .

Sol. Here $r = \frac{10.5}{2}$ cm = 5.25 cm

Inner surface area of the bowl = $2\pi r^2$

$$\begin{aligned}&= 2 \times \frac{22}{7} \times (5.25)^2 \text{ cm}^2 \\ &= 44 \times 0.75 \times 5.25 \text{ cm}^2 = 173.25 \text{ cm}^2\end{aligned}$$

Cost of tin plating 100 cm^2 = Rs 16

$$\text{Cost of tin plating } 173.25 \text{ cm}^2 = \text{Rs } \frac{16}{100} \times 173.25 = \mathbf{\text{Rs } 27.72} \text{ Ans.}$$

Q.6. Find the radius of a sphere whose surface area is 154 cm^2 .

Sol. Surface area of the sphere = $4\pi r^2$

$$\Rightarrow 154 = 4 \times \frac{22}{7} \times r^2$$

$$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22} = \frac{7 \times 7}{4}$$

$$\Rightarrow r = \frac{7}{2} = 3.5$$

Hence, radius of the sphere = **3.5 cm Ans.**

Q.7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Sol. Let diameter of the earth = $2r$

Then radius of the earth = r

$$\therefore \text{Diameter of the moon} = \frac{2r}{4} = \frac{r}{2}$$

$$\therefore \text{Radius of the moon} = \frac{r}{4}$$

$$\begin{aligned} \text{Now, surface area of the moon} &= 4\pi \left(\frac{r}{4}\right)^2 \\ &= \frac{\pi r^2}{4} \quad \dots \text{(i)} \end{aligned}$$

$$\text{Surface area of the earth} = 4\pi r^2 \quad \dots \text{(ii)}$$

$$\therefore \text{Required ratio} = \frac{\frac{\pi r^2}{4}}{4\pi r^2} = \frac{\pi r^2}{4 \times 4\pi r^2} = \frac{1}{16} = \mathbf{1 : 16 \text{ Ans.}}$$

Q.8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Sol. Inner radius of the bowl (r) = 5 cm

Thickness of the steel = 0.25 cm

\therefore Outer radius of the bowl (R) = (5 + 0.25) cm = 5.25 cm

Outer curved surface area of the bowl

$$= 2\pi R^2 = 2 \times \frac{22}{7} \times (5.25)^2 \text{ cm}^2 = \mathbf{173.25 \text{ cm}^2 \text{ Ans.}}$$

Q.9. A right circular cylinder just encloses a sphere of radius r (see figure). Find

- (i) surface area of the sphere,
- (ii) curved surface area of the cylinder,
- (iii) ratio of the areas obtained in (i) and (ii).

Sol. Here, radius of the sphere = r

Radius of the cylinder = r

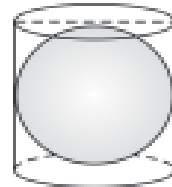
And, height of the cylinder = $2r$

(i) Surface area of the sphere = $4\pi r^2$

(ii) Curved surface area of the cylinder = $2\pi r h$

$$2\pi \times r \times 2r = 4\pi r^2 \text{ Ans.}$$

(iii) Required ratio = $\frac{4\pi r^2}{4\pi r^2} = \frac{1}{1}$



EXERCISE 13.5

Q.1. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes.

Sol. Here, $l = 4$ cm, $b = 2.5$ cm, $h = 1.5$ cm

Volume of 1 matchbox = lbh

$$= 4 \times 2.5 \times 1.5 \text{ cm}^3 = 15 \text{ cm}^3$$

Volume of 12 matchboxes = $15 \times 12 \text{ cm}^3 = \mathbf{180 \text{ cm}^3}$ **Ans.**

Q.2. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? ($1 \text{ m}^3 = 1000 \text{ l}$)

Sol. Here, $l = 6$ m, $b = 5$ m, $h = 4.5$ m

Volume of the tank = lbh

$$= 6 \times 5 \times 4.5 \text{ m}^3 = 135 \text{ m}^3$$

$$= 135 \times 1000 \text{ litres} = 1,35,000 \text{ litres.}$$

Hence, the tank can hold 1,35,000 litres of water. **Ans.**

Q.3. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Sol. Here, $l = 10$ m, $b = 8$ m, $h = ?$

Volume of the vessel = lbh

$$\Rightarrow 380 = 10 \times 8 \times h$$

$$\Rightarrow h = \frac{380}{10 \times 8} = 4.75$$

Hence, the tank must be made **4.75 m high** **Ans.**

Q.4. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs 30 per m^3 .

Sol. $l = 8$ m, $b = 6$ m, $h = 3$ m

Volume of the pit = lbh

$$= 8 \times 6 \times 3 \text{ m}^3 = 144 \text{ m}^3$$

Cost of digging $1 \text{ m}^3 = \text{Rs } 30$

$$\therefore \text{Cost of digging } 144 \text{ m}^3 = \text{Rs } 30 \times 144 = \mathbf{\text{Rs } 4320}$$
 Ans.

Q.5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Sol. Here, $l = 2.5$ m, $h = 10$ m, $b = ?$

$$\text{Capacity of the tank} = 50000 \text{ litres} = \frac{50000}{1000} \text{ m}^3 = 50 \text{ m}^3$$

Also, capacity of the tank = lbh

$$\Rightarrow 50 = 2.5 \times b \times 10 \quad \Rightarrow b = \frac{50}{25} = 2$$

Hence, breadth of the tank = **2 m** **Ans.**

Q.6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring $20\text{ m} \times 15\text{ m} \times 6\text{ m}$. For how many days will the water of this tank last?

Sol. Here, $l = 20\text{ m}$, $b = 15\text{ m}$, $h = 6\text{ m}$

Population of the village = 4000

Water consumed by 1 person in 1 day = 150 litres

\therefore Water consumed by 4000 persons in 1 day = 4000×150 litres

$$= \frac{4000 \times 150}{1000} \text{ m}^3 = 600 \text{ m}^3$$

Also, capacity of the tank = lbh

$$= 20 \times 15 \times 6 \text{ m}^3$$

$$\begin{aligned} \therefore \text{Required number of days} &= \frac{\text{Volume of the tank}}{\text{Water consumed in 1 day}} \\ &= \frac{20 \times 15 \times 6}{600} = 3 \end{aligned}$$

Hence, the water of this tank will last for 3 days. **Ans.**

Q.7. A godown measures $40\text{ m} \times 25\text{ m} \times 10\text{ m}$. Find the maximum number of wooden crates each measuring $1.5\text{ m} \times 1.25\text{ m} \times 0.5\text{ m}$ that can be stored in the godown.

Sol. Volume of the godown = $40 \times 25 \times 10 \text{ m}^3$

Volume 1 wooden crate = $1.5 \times 1.25 \times 0.5 \text{ m}^3$

\therefore Required number of crates = $\frac{\text{Volume of the godown}}{\text{Volume of 1 crate}}$

$$= \frac{40 \times 25 \times 10}{1.5 \times 1.25 \times 0.5} = 10666.67$$

Hence, the maximum number of wooden crates that can be stored in the godown = **10666 Ans.**

Q.8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Sol. Here, $a = 12\text{ cm}$

Volume of the cube = $a^3 = (12)^3 \text{ cm}^3 = 1728 \text{ cm}^3$

Now, volume of 1 smaller cube = $\frac{1728}{8} \text{ cm}^3 = 216 \text{ cm}^3$

Let side of the new cube be A .

Then $A^3 = 216$

$$\Rightarrow A = \sqrt[3]{216} = 6$$

Hence, side of the new cube = **6 cm Ans.**

Total surface area of the bigger cube = $6a^2$

$$= 6 \times (12)^2 \text{ cm}^2 = 6 \times 12 \times 12 \text{ cm}^2$$

Total surface area of 1 smaller cube = $6A^2$

$$= 6 \times 6^2 \text{ cm}^2 = 6 \times 6 \times 6 \text{ cm}^2$$

Hence, required ratio = $\frac{6 \times 12 \times 12}{6 \times 6 \times 6} = \frac{4}{1} = 4 : 1$ **Ans.**

Q.9. *A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour.
How much water will fall into the sea in a minute?*

Sol. Here, $b = 40$ m, $h = 3$ m, $l = 2$ km = 2000 m

Volume of water flowing through the river in 1 hour

$$= lbh = 2000 \times 40 \times 3 \text{ m}^3$$

\therefore Volume of water flowing through the river in 1 minute

$$= \frac{2000 \times 40 \times 3}{60} \text{ m}^3 = 4000 \text{ m}^3 \text{ Ans.}$$

EXERCISE 13.6

Q.1. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? ($1000 \text{ cm}^3 = 1\text{l}$)

Sol. Here, $h = 25 \text{ cm}$, $2\pi r = 132 \text{ cm}$.

$$2\pi r = 132$$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 132$$

$$\Rightarrow r = \frac{132 \times 7}{2 \times 22} \text{ cm} = 21 \text{ cm}$$

$$\begin{aligned} \text{Volume of the cylinder} &= \pi r^2 h = \frac{22}{7} \times 21 \times 21 \times 25 \text{ cm}^3 \\ &= 34650 \text{ cm}^3 \\ &= \frac{34650}{1000} \text{ litres} = \mathbf{34.65 \text{ litres}} \text{ Ans.} \end{aligned}$$

Q.2. The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.

Sol. Here, inner radius (r) = $\frac{24}{2} \text{ cm} = 12 \text{ cm}$

$$\text{Outer radius (R)} = \frac{28}{2} \text{ cm} = 14 \text{ cm}, h = 35 \text{ cm}$$

$$\begin{aligned} \text{Volume of the wood used in the pipe} &= \pi(R^2 - r^2) h \\ &= \frac{22}{7} [(14)^2 - (12)^2] \times 35 \text{ cm}^3 \\ &= \frac{22}{7} \times 26 \times 2 \times 35 \text{ cm}^3 = 5720 \text{ cm}^3 \end{aligned}$$

Mass of 1 cm^3 of wood = 0.6 g

$$\therefore \text{Mass of } 5720 \text{ cm}^3 \text{ of wood} = 0.6 \times 5720 \text{ g} = 3432 \text{ g} = \mathbf{3.432 \text{ kg}} \text{ Ans.}$$

Q.3. A soft drink is available in two packs — (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Sol. For tin can with rectangular base.

$$l = 5 \text{ cm}, b = 4 \text{ cm}, h = 15 \text{ cm}$$

$$\text{Volume of the tin can} = lbh = 5 \times 4 \times 15 \text{ cm}^3 = 300 \text{ cm}^3$$

For plastic cylinder with circular base.

$$r = \frac{7}{2} \text{ cm} = 3.5 \text{ cm}, h = 10 \text{ cm}$$

$$\begin{aligned} \text{Volume of the plastic cylinder} &= \pi r^2 h \\ &= \frac{22}{7} \times 3.5 \times 3.5 \times 10 \text{ cm}^3 = 385 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Difference in the capacities of the two containers} \\ &= (385 - 300) \text{ cm}^3 = 85 \text{ cm}^3 \end{aligned}$$

Hence, the plastic cylinder with circular base has greater capacity by 85 cm^3 **Ans.**

Q.4. If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm , then find (i) radius of its base (ii) its volume (Use $\pi = 3.14$)

Sol. Here, $h = 5 \text{ cm}$, $2\pi rh = 94.2 \text{ cm}^2$.

$$(i) \quad 2\pi rh = 94.2 \\ \Rightarrow 2 \times 3.14 \times r \times 5 = 94.2$$

$$\Rightarrow r = \frac{94.2}{2 \times 3.14 \times 5} = 3$$

Hence, base radius of the cylinder = 3 cm **Ans.**

$$(ii) \quad \text{Volume of the cylinder} = \pi r^2 h \\ = 3.14 \times 3 \times 3 \times 5 \text{ cm}^3 = 141.3 \text{ cm}^3 \text{ **Ans.**}$$

Q.5. It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of Rs 20 per m^2 , find

- (i) Inner curved surface area of the vessel,
(ii) radius of the base,
(iii) capacity of the vessel.

Sol. Here, $h = 10 \text{ m}$

$$(i) \quad \text{Inner curved surface area} = \frac{\text{Total cost}}{\text{Cost of painting per m}^2} \\ = \frac{2200}{20} \text{ m}^2 = 110 \text{ m}^2 \text{ **Ans.**}$$

$$(ii) \quad \text{We have, } 2\pi rh = 110 \\ \Rightarrow 2 \times \frac{22}{7} \times r \times 10 = 110 \\ \Rightarrow r = \frac{110 \times 7}{2 \times 22 \times 10} = 1.75 \text{ m **Ans.**}$$

$$(iii) \quad \text{Capacity of the vessel} = \pi r^2 h \\ = \frac{22}{7} \times 1.75 \times 1.75 \times 10 \text{ m}^3 = 96.25 \text{ m}^3 \\ = 96.25 \text{ kl **Ans.** } [1 \text{ m}^3 = 1 \text{ kl}]$$

Q.6. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres . How many square metres of metal sheet would be needed to make it?

Sol. Here, $h = 1 \text{ m}$, volume = 15.4 litres

$$= \frac{15.4}{1000} \text{ m}^3 = 0.0154 \text{ m}^3$$

Also, volume of the cylindrical vessel = $\pi r^2 h$

$$\Rightarrow 0.0154 = \frac{22}{7} \times r^2 \times 1 \\ \Rightarrow r^2 = \frac{0.0154 \times 7}{22} = 0.0049$$

$$\Rightarrow r = 0.07 \text{ m}$$

$$\therefore \text{Total surface area of the cylinder} = 2\pi r (h + r)$$

$$= 2 \times \frac{22}{7} \times 0.07 (1 + 0.07) \text{ m}^2$$

$$= 44 \times 0.01 \times 1.07 \text{ m}^2 = 0.4708 \text{ m}^2$$

Hence, 0.4708 m^2 of metal sheet would be needed **Ans.**

Q.7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

Sol. Here, $h = 14 \text{ cm}$.

$$\text{Radius of the pencil (R)} = \frac{7}{2} \text{ mm} = 0.35 \text{ cm.}$$

$$\text{Radius of the graphite (r)} = \frac{1}{2} \text{ mm} = 0.05 \text{ cm.}$$

$$\text{Volume of the the graphite} = \pi r^2 h$$

$$= \frac{22}{7} \times 0.05 \times 0.05 \times 14 \text{ cm}^3 = 0.11 \text{ cm}^3$$

$$\text{Volume of the the wood} = \pi (R^2 - r^2)h$$

$$= \frac{22}{7} \times [(0.35)^2 - (0.05)^2] \times 14 \text{ cm}^3$$

$$= \frac{22}{7} \times 0.4 \times 0.3 \times 14 \text{ cm}^3 = 5.28 \text{ cm}^3$$

Hence, volume of the wood = **5.28 cm³** and volume of the graphite
= **0.11 cm³ Ans.**

Q.8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

Sol. Here, $r = \frac{7}{2} \text{ cm} = 3.5 \text{ cm}$, $h = 4 \text{ cm}$

$$\text{Capacity of 1 cylindrical bowl} = \pi r^2 h$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 4 \text{ cm}^3 = 154 \text{ cm}^3$$

Hence, soup consumed by 250 patients per day

$$= 250 \times 154 \text{ cm}^3 = \mathbf{38500 \text{ cm}^3 \text{ Ans.}}$$

EXERCISE 13.7

Q.1. Find the volume of the right circular cone with

- (i) radius 6 cm, height 7 cm (ii) radius 3.5 cm, height 12 cm

Sol. (i) Here, $r = 6$ cm, $h = 7$ cm

$$\begin{aligned}\text{Volume of the cone} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7 \text{ cm}^3 = \mathbf{264 \text{ cm}^3 \text{ Ans.}}\end{aligned}$$

(ii) Here, $r = 3.5$ cm, $h = 12$ cm

$$\begin{aligned}\text{Volume of the cone} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12 \text{ cm}^3 = \mathbf{154 \text{ cm}^3 \text{ Ans.}}\end{aligned}$$

Q.2. Find the capacity in litres of a conical vessel with

- (i) radius 7 cm, slant height 25 cm
(ii) height 12 cm, slant height 13 cm

Sol. (i) Here, $r = 7$ cm, $l = 25$ cm

$$\therefore r = \sqrt{l^2 - r^2} = \sqrt{625 - 49} = \sqrt{576} = 24 \text{ cm.}$$

$$\begin{aligned}\text{Volume of the conical vessel} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 \text{ cm}^3 = 1232 \text{ cm}^3 \\ &= \frac{1232}{1000} \text{ litres} = \mathbf{1.232 \text{ litres Ans.}}\end{aligned}$$

(ii) Here, $h = 12$ cm, $l = 13$ cm

$$\therefore h = \sqrt{l^2 - h^2} = \sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5 \text{ cm}$$

$$\begin{aligned}
 \text{Volume of the conical vessel} &= \frac{1}{3} \pi r^2 h \\
 &= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 \text{ cm}^3 = \frac{22 \times 5 \times 5 \times 4}{7} \text{ cm}^3 \\
 &= \frac{22 \times 5 \times 5 \times 4}{7 \times 1000} \text{ litres} = \frac{11}{35} \text{ litres Ans.}
 \end{aligned}$$

Q.3. *The height of a cone is 15 cm. If its volume is 1570 cm³, find the radius of the base. (Use $\pi = 3.14$)*

Sol. (i) Here, $h = 15$ cm, volume = 1570 cm³

$$\begin{aligned}
 \text{Volume of the cone} &= \frac{1}{3} \pi r^2 h \\
 \Rightarrow 1570 &= \frac{1}{3} \times 3.14 \times r^2 \times 15 \\
 \Rightarrow r^2 &= \frac{1570 \times 3}{3.14 \times 15} = 100 \\
 \Rightarrow r &= 10
 \end{aligned}$$

Hence, radius of the base = **10 cm Ans.**

Q.4. *If the volume of a right circular cone of height 9 cm is 48π cm³, find the diameter of its base.*

Sol. Here, $h = 9$ cm, volume = 48π cm³

$$\begin{aligned}
 \text{Volume of the cone} &= \frac{1}{3} \pi r^2 h \\
 \Rightarrow 48\pi &= \frac{1}{3} \pi \times r^2 \times 9 \\
 \Rightarrow r^2 &= \frac{48 \pi \times 3}{\pi \times 9} = 16 \\
 \Rightarrow r &= 4
 \end{aligned}$$

Hence, base diameter of the cone = 2×4 cm = **8 cm Ans.**

Q.5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

Sol. Here, $r = \frac{3.5}{2}$ m = 1.75 m, $h = 12$ m

$$\begin{aligned}\text{Capacity of the pit} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 12 \text{ m}^3 \\ &= 38.5 \text{ m}^3 = \mathbf{38.5 \text{ kl Ans.}}\end{aligned}$$

Q.6. The volume of a right circular cone is 9856 cm^3 . If the diameter of the base is 28 cm, find

- (i) height of the cone
- (ii) slant height of the cone
- (iii) curved surface area of the cone.

Sol. Here, $r = \frac{28}{2}$ cm = 14 cm, volume = 9856 cm^3

$$\begin{aligned}\text{(i) Volume of the cone} &= \frac{1}{3} \pi r^2 h \\ \Rightarrow 9856 &= \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h \\ \Rightarrow h &= \frac{9856 \times 3 \times 7}{22 \times 14 \times 14} = 48\end{aligned}$$

Hence, height of the cone = 48 cm **Ans.**

$$\begin{aligned}\text{(ii) Slant height } l &= \sqrt{h^2 + r^2} = \sqrt{(48)^2 + (14)^2} \\ &= \sqrt{2304 + 196} = \sqrt{2500} = 50\end{aligned}$$

Hence, slant height of the cone = 50 cm **Ans.**

$$\begin{aligned}\text{(iii) Curved surface area of the cone} &= \pi r l \\ &= \frac{22}{7} \times 14 \times 50 \text{ cm}^2 = \mathbf{2200 \text{ cm}^2 \text{ Ans.}}\end{aligned}$$

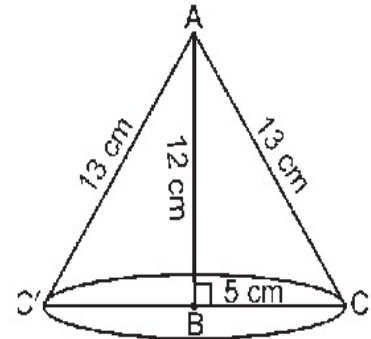
Q.7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Sol. The solid formed is a cone, whose height

$h = 12$ cm, base radius $r = 5$ cm.

$$\therefore \text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 5 \times 5 \times 12 \text{ cm}^3 = 100 \pi \text{ cm}^3 \text{ Ans.}$$



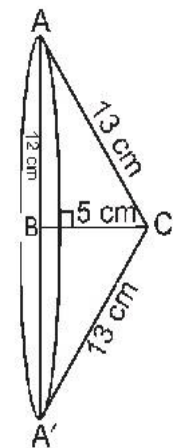
Q.8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in questions 7 and 8.

Sol. Here radius r of the cone = 12 cm and height h of the cone = 5 cm.

$$\therefore \text{Volume of the cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \times 12 \times 12 \times 5 = 240 \pi \text{ cm}^3 \text{ Ans.}$$

$$\text{Hence, required ratio} = \frac{100 \pi}{240 \pi} = \frac{5}{12} = 5 : 12 \text{ Ans.}$$



Q.9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Sol. Here, radius $r = \frac{10.5}{2}$ m = 5.25 m, $h = 3$ m

$$\begin{aligned}\text{Volume of the heap} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3 \text{ m}^3 = \mathbf{86.625 \text{ m}^3 \text{ Ans.}}\end{aligned}$$

$$\begin{aligned}\text{Now, } l &= \sqrt{h^2 + r^2} = \sqrt{3^2 + (5.25)^2} \\ &= \sqrt{9 + 27.5625} = \sqrt{36.5625} = 6.05 \text{ m (approx)}\end{aligned}$$

$$\begin{aligned}\text{Curved surface area of the cone} &= \pi r l \\ &= \frac{22}{7} \times 5.25 \times 6.05 \text{ m}^2 = 99.825 \text{ m}^2\end{aligned}$$

Hence, 99.825 m² of canvas is needed. **Ans.**

EXERCISE 13.8

Q.1. Find the volume of a sphere whose radius is

- (i) 7 cm (ii) 0.63 m

Sol. (i) Here, $r = 7$ cm

$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{ cm}^3 = 1437\frac{1}{3} \text{ cm}^3 \text{ Ans.}$$

(ii) Here, $r = 0.63$ m

$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.63)^3 \text{ m}^3 = 1.05 \text{ m}^3 \text{ (approx) Ans.}$$

Q.2. Find the amount of water displaced by a solid spherical ball of diameter

- (i) 28 cm (ii) 0.21 m

Sol. (i) Here, $r = \frac{28}{2}$ cm = 14 cm

Volume of water displaced by the spherical ball

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 \text{ cm}^3$$

$$= 11498\frac{2}{3} \text{ cm}^3 \text{ Ans.}$$

(ii) Here, $r = \frac{0.21}{2}$ m = 0.105 m

Volume of the water displaced by the spherical ball

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 0.105 \times 0.105 \times 0.105 \text{ m}^3$$

$$= 0.004851 \text{ m}^3 \text{ Ans.}$$

Q.3. *The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm³?*

Sol. Here, $r = \frac{4.2}{2}$ cm = 2.1 cm

$$\begin{aligned}\text{Volume of the ball} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 \text{ cm}^3 \\ &= 38.808 \text{ cm}^3\end{aligned}$$

$$\text{Density of the metal} = 8.9 \text{ g/cm}^3$$

$$\therefore \text{Mass of the ball} = 8.9 \times 38.808 \text{ g}$$

$$= \mathbf{345.39 \text{ g (approx) Ans.}}$$

Q.4. *The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?*

Sol. Let diameter of the earth be $2r$.

Then radius of the earth = r

$$\text{So, diameter of the moon} = \frac{2r}{4} = \frac{r}{2}$$

$$\Rightarrow \text{Radius of the moon} = \frac{r}{4}$$

$$\text{Volume of the earth} = \frac{4}{3}\pi r^3 \quad \dots \text{ (i)}$$

$$\text{Volume of the moon} = \frac{4}{3}\pi \left(\frac{r}{4}\right)^3 \quad \dots \text{ (ii)}$$

$$\frac{\text{Volume of the earth}}{\text{Volume of the moon}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi \left(\frac{r}{4}\right)^3} \quad [\text{From (i) and (ii)}]$$

$$\frac{r^3}{\frac{r^3}{64}} = \frac{64}{1} = 64 \text{ Ans.}$$

\Rightarrow Volume of the moon = $\frac{1}{64}$ \times volume of the earth

Hence, volume of the moon is $\frac{1}{64}$ of volume of the earth. **Ans.**

Q.5. *How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?*

Sol. Here, $r = \frac{10.5}{2}$ cm = 5.25 cm

$$\begin{aligned} \text{Volume of the hemispherical bowl} &= \frac{2}{3} \pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 5.25 \text{ cm}^3 = 303 \text{ cm}^3 \text{ (approx)} \end{aligned}$$

Hence, the hemispherical bowl can hold $\frac{303}{1000}$ litres = **0.303 liters of milk. Ans.**

Q.6. *A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.*

Sol. Here, inner radius of the tank (r) = 1 m

Thickness of the iron sheet = 1 cm = 0.01 m

\therefore External radius of the tank (R) = (1 + 0.01) m = 1.01 m

Volume of the iron used to make the tank

$$\begin{aligned} &= \frac{2}{3} \pi (R^3 - r^3) \\ &= \frac{2}{3} \times \frac{22}{7} \times [(1.01)^3 - 1^3] \text{ m}^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times 0.030301 \text{ m}^3 \\ &= \mathbf{0.06348 \text{ m}^3 \text{ Ans.}} \end{aligned}$$

Q.7. Find the volume of a sphere whose surface area is 154 cm^2 .

Sol. Here, $4\pi r^2 = 154$

$$\Rightarrow r^2 = 4 \times \frac{22}{7} \times r^2 = 154$$

$$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4}$$

$$\Rightarrow r = \frac{7}{2} = 3.5 \text{ cm}$$

$$\therefore \text{Volume of the sphere} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 \text{ cm}^3 = 179 \frac{2}{3} \text{ cm}^3 \text{ Ans.}$$

Q.8. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of Rs 498.96. If the cost of white-washing is Rs 2.00 per square metre, find the

- (i) inside surface area of the dome,
- (ii) volume of the air inside the dome.

Sol. (i) Inner surface of the dome

$$\begin{aligned} &= \frac{\text{Total cost}}{\text{Cost of white washing per m}^2} \\ &= \frac{498.96}{2} \text{ m}^2 = 249.48 \text{ m}^2 \text{ Ans.} \end{aligned}$$

(ii) Let radius of the dome be r m.

$$\text{Then } 2\pi r^2 = 249.48$$

$$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 249.48$$

$$\Rightarrow r^2 = \frac{249.48 \times 7}{2 \times 22} = 39.69$$

$$\Rightarrow r = 6.3 \text{ cm}$$

$$\begin{aligned}\therefore \text{Volume of the air inside the dome} &= \frac{2}{3} \pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3 \text{ m}^3 = \mathbf{523.9 \text{ m}^3 \text{ Ans.}}\end{aligned}$$

Q.9. *Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S' . Find the*

(i) radius r' of the new sphere.

(ii) ratio of S and S' .

Sol. (i) Volume of a sphere of radius $r = \frac{4}{3} \pi r^3$

$$\therefore \text{Volume of 27 such spheres} = 27 \times \frac{4}{3} \pi r^3 = 36\pi r^3$$

$$\text{Volume of the sphere with radius } r' = \frac{4}{3} \pi r'^3$$

$$\therefore 36\pi r^3 = \frac{4}{3} \pi r'^3$$

$$\Rightarrow 27r^3 = r'^3$$

$$\Rightarrow r' = \sqrt[3]{27r^3}$$

$$\Rightarrow r' = \mathbf{3r \text{ Ans.}}$$

(ii) Surface area (S) of the sphere with radius $r = 4\pi r^2$

$$\begin{aligned}\text{Surface area (S')} \text{ of the sphere with radius } r' &= 4\pi r'^2 \\ &= 4\pi (3r)^2 = 36\pi r^2\end{aligned}$$

$$\therefore \frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = \mathbf{1 : 9 \text{ Ans.}}$$

Q.10. *A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?*

Sol. Here, $r = \frac{3.5}{2} \text{ mm} = 1.75 \text{ mm}$

$$\text{Volume of the capsule} = \frac{4}{3} \pi r^3$$

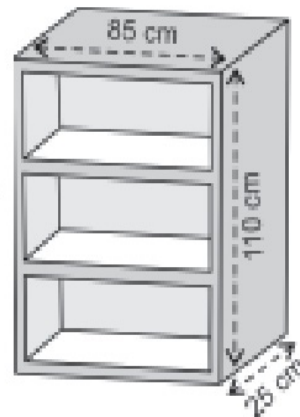
$$= \frac{4}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 1.75 \text{ mm}^3$$

$$= 22.46 \text{ mm}^3 \text{ (approx)}$$

Hence, 22.46 mm^3 of medicine is needed to fill the capsule **Ans.**

EXERCISE 13.9 (Optional)

Q.1. A wooden bookshelf has external dimensions as follows : Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see figure). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2 , find the total expenses required for polishing and painting the surface of the bookshelf.



Sol. Here, external dimensions of the bookshelf are :

$$L = 110 \text{ cm}, B = 85 \text{ cm}, H = 25 \text{ cm}$$

$$\text{Thickness of the plank} = 5 \text{ cm}$$

Internal dimensions of the bookshelf are :

$$l = (110 - 5 - 5) \text{ cm} = 100 \text{ cm},$$

$$b = (85 - 5 - 5) \text{ cm} = 75 \text{ cm},$$

$$h = (25 - 5) \text{ cm} = 20 \text{ cm}$$

External surface area of the bookshelf

$$\begin{aligned} &= LB + 2(BH + HL) \\ &= 110 \times 85 \text{ cm}^2 + 2(85 \times 25 + 25 \times 110) \text{ cm}^2 \\ &= (9350 + 9750) \text{ cm}^2 = 19100 \text{ cm}^2 \end{aligned}$$

Surface area of the border

$$\begin{aligned} &= (4 \times 75 \times 5 + 110 \times 5 \times 2) \text{ cm}^2 \\ &= (1500 + 1100) \text{ cm}^2 = 2600 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Total surface area to be polished} &= (19100 + 2600) \text{ cm}^2 \\ &= 21700 \text{ cm}^2 \end{aligned}$$

$$\therefore \text{Cost of polishing the outer surface} = \text{Rs } \frac{21700 \times 20}{100} = \text{Rs } 4340 \quad \dots \text{ (i)}$$

$$\begin{aligned}
 \text{Inner surface area of the bookshelf} &= lb + 2(bh + hl) \\
 &= 100 \times 75 \text{ cm}^2 + 2(75 \times 20 + 100) \text{ cm}^2 \\
 &= 7500 \text{ cm}^2 + 2(1500 + 2000) \text{ cm}^2 \\
 &= (7500 + 7000) \text{ cm}^2 = 14500 \text{ cm}^2
 \end{aligned}$$

$$\text{Surface area of the two racks} = 4 \times 75 \times 20 \text{ cm}^2 = 6000 \text{ cm}^2$$

$$\begin{aligned}
 \text{Inner surface area covered by the racks} &= (75 \times 5 \times 2 + 20 \times 5 \times 4) \text{ cm}^2 \\
 &= (750 + 400) \text{ cm}^2 = 1150 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Total surface area to be painted} &= (14500 + 6000 - 1150) \text{ cm}^2 \\
 &= 19350 \text{ cm}^2
 \end{aligned}$$

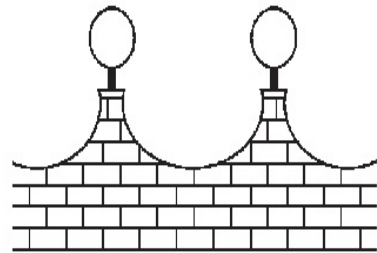
$$\therefore \text{Cost of painting the inner surface} = \text{Rs } \frac{19350 \times 10}{100} = \text{Rs } 1935 \quad \dots \text{ (ii)}$$

From (i), and (ii), we have,

Total expenses required for polishing and painting the surface of the bookshelf.

$$= \text{Rs } (4340 + 1935) = \text{Rs } 6275 \text{ Ans.}$$

Q.2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in the figure. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2 .



Sol. Radius of a sphere = $\frac{21}{2}$ cm = 10.5 cm.

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5 \text{ cm}^2 = 1386 \text{ cm}^2$$

Area of the base of the cylinder (support) = πR^2

$$= \pi \times (1.5)^2 = \frac{22}{7} \times 1.5 \times 1.5 \text{ cm}^2$$

$$= 7.07 \text{ cm}^2$$

Area of a sphere to painted silver

$$= (1386 - 7.07) \text{ cm}^2$$

$$= 1378.93 \text{ cm}^2$$

Area of spheres to be painted silver = $8 \times 1378.93 \text{ cm}^2$

$$\therefore \text{cost of painting the spheres} = \text{Rs } \frac{8 \times 1378.93 \times 25}{100}$$

$$= \text{Rs } 2757.86$$

Curved surface area of a cylinder (support)

$$= 2 \times \frac{22}{7} \times 1.5 \times 7 \text{ cm}^2$$

Curved surface area of 8 supports = $8 \times 2 \times \frac{22}{7} \times 1.5 \times 7 \text{ cm}^2$

Cost of painting the supports = $\text{Rs } 8 \times 2 \times \frac{22}{7} \times 1.5 \times 7 \times \frac{5}{100}$

$$= \text{Rs } 26.40$$

Total cost required of paint = $\text{Rs } (2757.86 + 26.40) = \text{Rs } 2784.26 \text{ Ans.}$

Q.3. *The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?*

Sol. Let originally the diameter of the sphere be $2r$.

Then, radius of the sphere = r

Surface area of the sphere = $4\pi r^2$... (i)

$$\text{New diameter of the sphere} = 2r - 2r \times \frac{25}{100} = \frac{3r}{2}$$

$$\therefore \text{New radius of the sphere} = \frac{3r}{4}$$

$$\text{Surface area of the new sphere} = 4\pi \left(\frac{3r}{4}\right)^2 = \frac{9\pi r^2}{4}$$

$$\text{Decrease in surface area} = 4\pi r^2 - \frac{9\pi r^2}{4} = \frac{7\pi r^2}{4}$$

$$\text{Per cent decrease} = \frac{\frac{7\pi r^2}{4} \times 100}{4\pi r^2} = \frac{7}{16} \times 100 = \frac{175}{4} = 43.75$$

Hence, the surface area decreases by **43.75% Ans.**